Short description

The celestial three body problem consists of three masses moving in the gravitational field.

In the **restricted** problem, one only considers the motion in two dimensions and lets the mass of one body (the satellite) go to zero. This problem still is chaotic and therefore is not integrable.

The equations of motion for the satellite (r_3) in the inertial system are:

$$m_3 * \ddot{\vec{r}}_3 = - \frac{m_3 * m_2}{\left(\left|\vec{r}_3 - \vec{r}_2\right|\right)^3} \left(\vec{r}_3 - \vec{r}_2\right) - \frac{m_3 * m_1}{\left(\left|\vec{r}_3 - \vec{r}_1\right|\right)^3} \left(\vec{r}_3 - \vec{r}_1\right)$$

$$r_1(t) = \left(r_{1x}(t), r_{1y}(t) \right) \quad r_2(t) = \left(r_{2x}(t), r_{2y}(t) \right)$$

In the limit (m_3 -> 0) $r_1(t)$, $r_2(t)$ become independent of the satellite and the planets follow circles around their center of mass. Let m1 + m2 = 1, their distance d becomes 1 and the angular velocity becomes 1 too. After a transformation to the system, which rotates with the two planets with the center of mass in the origin, one gets the equations of motion for the satellite:

$$\ddot{x} - 2\dot{y} = \frac{\partial}{\partial x} \Omega(x,y) \qquad \ddot{y} + 2\dot{x} = \frac{\partial}{\partial y} \Omega(x,y)$$

$$\Omega(x,y) = \frac{1}{2}\left(x^2 + y^2\right) + U(x,y)$$

$$U(x,y) = \frac{1 - \mu}{\sqrt{(x + \mu)^2 + y^2}} + \frac{\mu}{\sqrt{(x - 1 + \mu)^2 + y^2}}$$

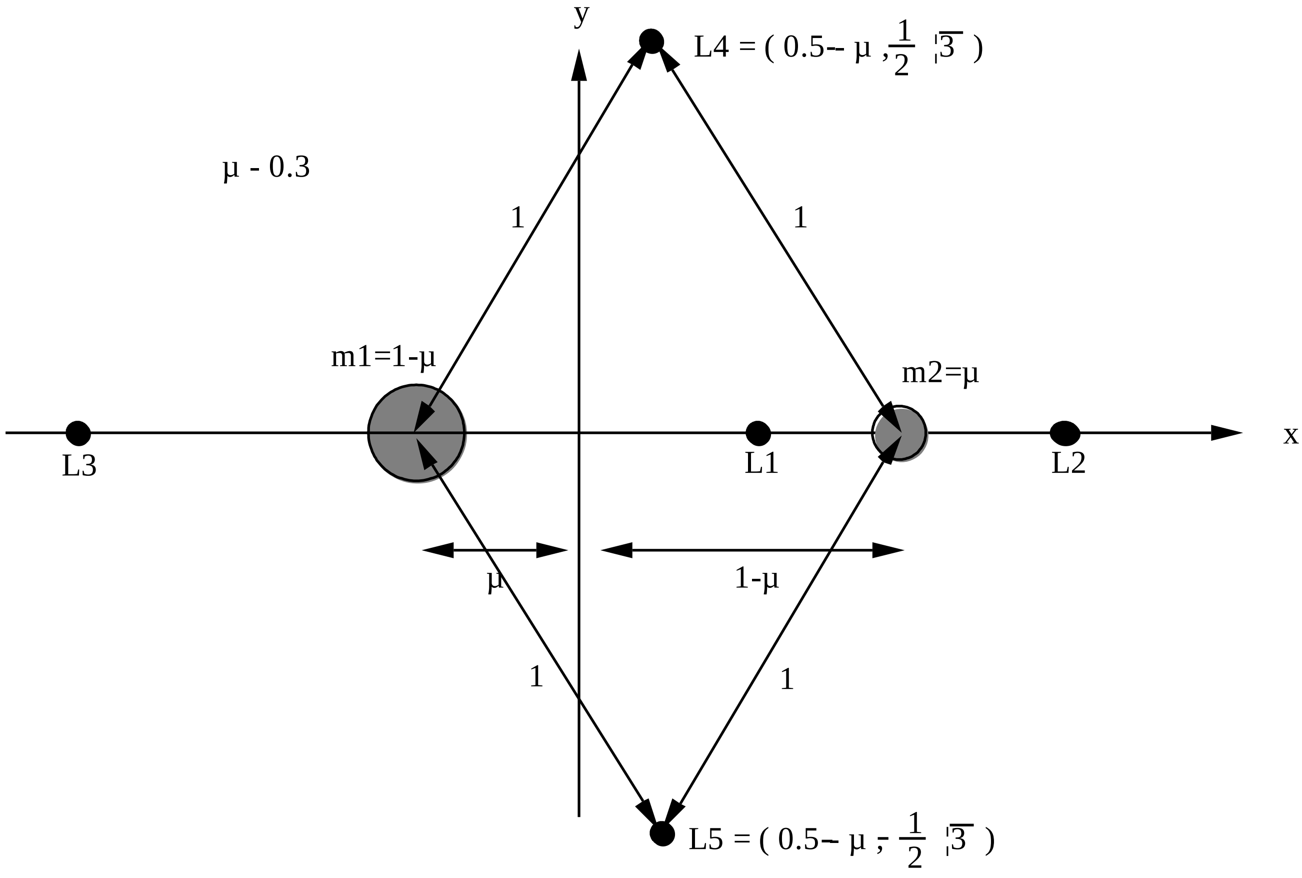
m1 + m2 = 1 i.e. m1 := 1-μ , m2 = μ for $0<\mu<1$

$\vec{r}_1 = (-\mu, 0)$, $\vec{r}_2 = (1-\mu, 0)$

Conservation of energy gives one invariant (Jacobi-Integral):

$$v^2 = 2\Omega(x,y) - \text{const}$$

All points (x,y) with zero speed (0 = 2 Ω(x,y) + const) are called Hill's curves. The satellite can't cross these. There are 5 fixed points L1 - L5 in the rotating system, see grafic below. L4 and L5 are stable for ($0 < \mu < 0.03852$). L1, L4 always are instable.


y
L4 = (0.5– μ , 1/2 √3)
μ - 0.3
1
1
m1=1-μ
m2=μ
x
L3
L1
L2
μ
1-μ
1
1
L5 = (0.5– μ , - 1/2 √3)